查表範例 (I)

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- 標準常態分配表 (cumulative standard normal distribution table)
- t 分配表 (t distribution table)
- 二項分配表 (cumulative Binomial distribution table)
- 卜瓦松分配表 (cumulative Poisson distribution table)

附表 4　標準常態分配

z	0.00	0.01	0.02	0.03	0.04	z
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.0
0.1	0.53983	0.54379	0.54776	0.55172	0.55567	0.1
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.2
0.3	0.61791	0.62172	0.62551	0.62930	0.63307	0.3
0.4	0.65542	0.65910	0.66273	0.66640	0.67003	0.4
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.5
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.6
0.7	0.75803	0.75115	0.76424	0.76730	0.77035	0.7
0.8	0.78814	0.79103	0.79389	0.79673	0.79954	0.8
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.9
1.0	0.84134	0.84375	0.84613	0.84849	0.85083	1.0
1.1	0.86433	0.86650	0.86864	0.87076	0.87285	1.1
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	1.2
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	1.3
1.4	0.91924	0.92073	0.92219	0.92364	0.92506	1.4
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	1.5
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	1.6
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	1.7
1.8	0.96407	0.96485	0.96562	0.96637	0.96711	1.8
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	1.9
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	2.0
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	2.1
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	2.2
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	2.3
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	2.4
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	2.5
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	2.6
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	2.7
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	2.8
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	2.9
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	3.0
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	3.1
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	3.2
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	3.3
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	3.4
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	3.5
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	3.6
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	3.7
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	3.8
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	3.9

附表 4　標準常態分配 (續)

z	0.05	0.06	0.07	0.08	0.09	z
0.0	0.51994	0.52392	0.52790	0.53188	0.53586	0.0
0.1	0.55962	0.56356	0.56749	0.57142	0.57534	0.1
0.2	0.59871	0.60257	0.60642	0.61026	0.61409	0.2
0.3	0.63683	0.64058	0.64431	0.64803	0.65173	0.3
0.4	0.67364	0.67724	0.68082	0.68438	0.68793	0.4
0.5	0.70884	0.71226	0.71566	0.71904	0.72240	0.5
0.6	0.74215	0.74537	0.74857	0.75175	0.75490	0.6
0.7	0.77337	0.77637	0.77935	0.78230	0.78523	0.7
0.8	0.80234	0.80510	0.80785	0.81057	0.81327	0.8
0.9	0.82894	0.83147	0.83397	0.83646	0.83891	0.9
1.0	0.85314	0.85543	0.85769	0.85993	0.86214	1.0
1.1	0.87493	0.87697	0.87900	0.88100	0.88297	1.1
1.2	0.89435	0.89616	0.89796	0.89973	0.90147	1.2
1.3	0.91149	0.91308	0.91465	0.91621	0.91773	1.3
1.4	0.92647	0.92785	0.92922	0.93056	0.93189	1.4
1.5	0.93943	0.94062	0.94179	0.94295	0.94408	1.5
1.6	0.95053	0.95154	0.95254	0.95352	0.95448	1.6
1.7	0.95994	0.96080	0.96164	0.96246	0.96327	1.7
1.8	0.96784	0.96856	0.96926	0.96995	0.97062	1.8
1.9	0.97441	0.97500	0.97558	0.97615	0.97670	1.9
2.0	0.97982	0.98030	0.98077	0.98124	0.98169	2.0
2.1	0.98422	0.98461	0.98500	0.98537	0.98574	2.1
2.2	0.98778	0.98809	0.98840	0.98870	0.98899	2.2
2.3	0.99061	0.99086	0.99111	0.99134	0.99158	2.3
2.4	0.99286	0.99305	0.99324	0.99343	0.99361	2.4
2.5	0.99461	0.99477	0.99492	0.99506	0.99520	2.5
2.6	0.99598	0.99609	0.99621	0.99632	0.99643	2.6
2.7	0.99702	0.99711	0.99720	0.99728	0.99736	2.7
2.8	0.99781	0.99788	0.99795	0.99801	0.99807	2.8
2.9	0.99841	0.99846	0.99851	0.99856	0.99861	2.9
3.0	0.99886	0.99889	0.99893	0.99897	0.99900	3.0
3.1	0.99918	0.99921	0.99924	0.99926	0.99929	3.1
3.2	0.99942	0.99944	0.99946	0.99948	0.99950	3.2
3.3	0.99960	0.99961	0.99962	0.99964	0.99965	3.3
3.4	0.99972	0.99973	0.99974	0.99975	0.99976	3.4
3.5	0.99981	0.99981	0.99982	0.99983	0.99983	3.5
3.6	0.99987	0.99987	0.99988	0.99988	0.99989	3.6
3.7	0.99991	0.99992	0.99992	0.99992	0.99992	3.7
3.8	0.99994	0.99994	0.99995	0.99995	0.99995	3.8
3.9	0.99996	0.99996	0.99996	0.99997	0.99997	3.9

標準常態分配

假設 Φ 為標準常態分配之累積分布函數 (cumulative distribution function)。Φ(Z) 代表小於 Z 之機率。表格型態:

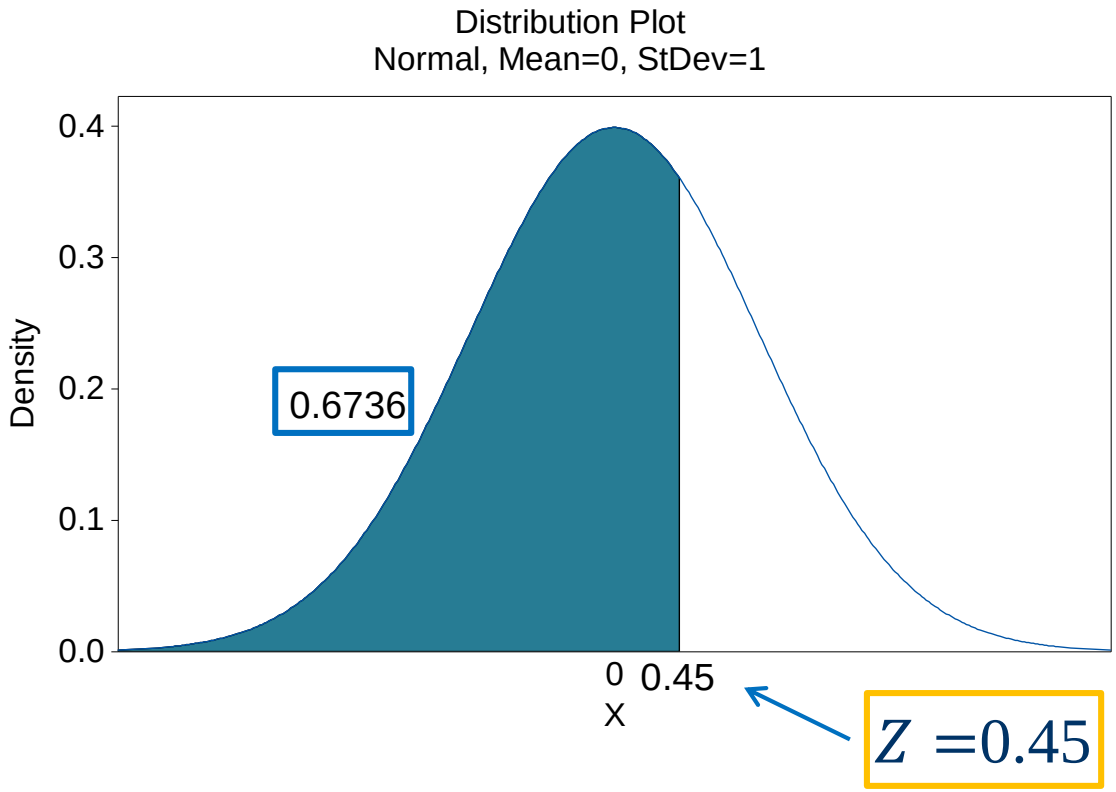
- cumulative from mean (0 to Z) (標準常態分配$\mu = 0, \sigma = 1$)
- cumulative (less than Z) (一般常用)
- complementary cumulative 1 – Φ(Z)

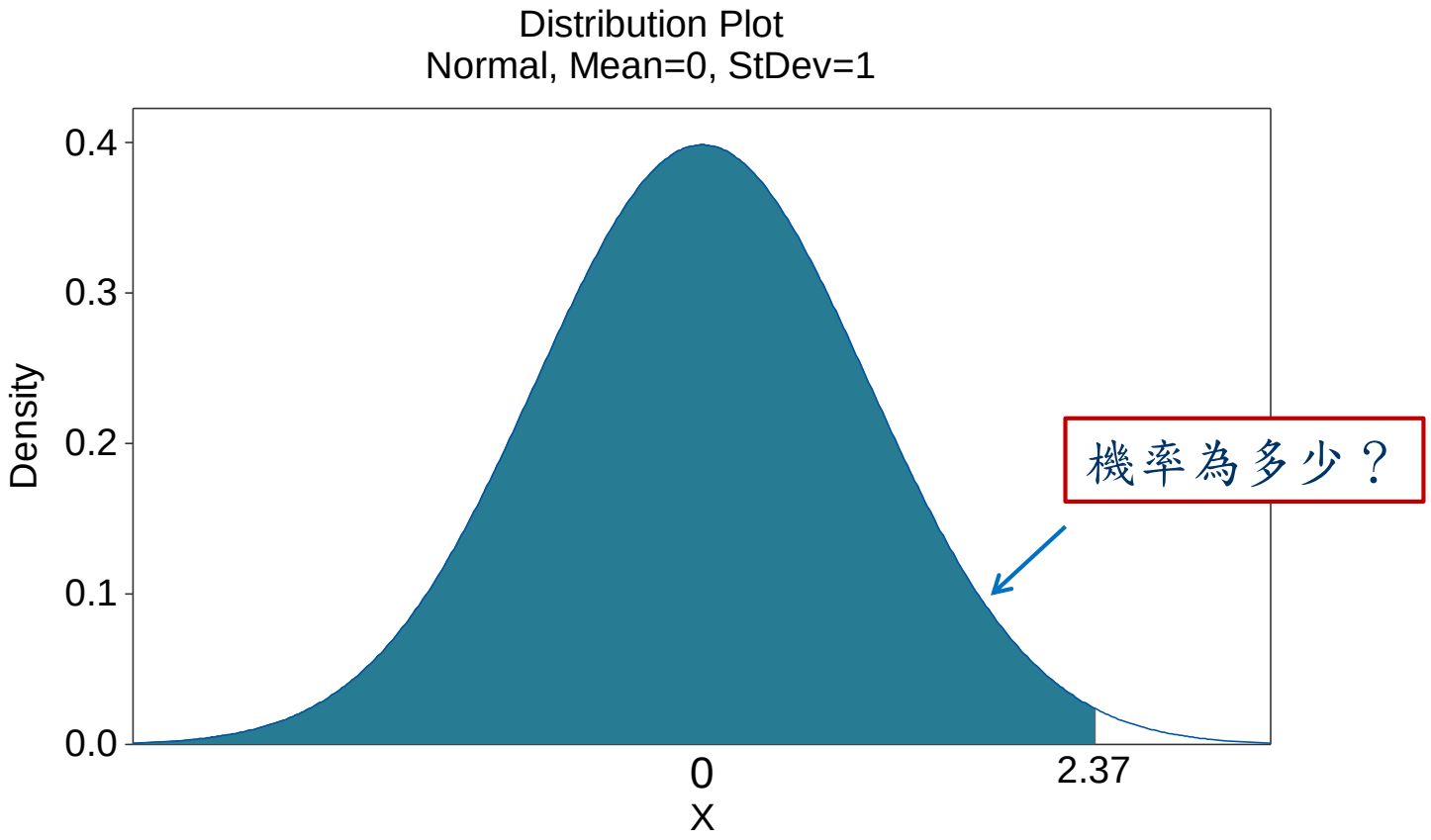
一般表格為 Φ(Z) (Z 為正號，極少數教科書會給正、負號)

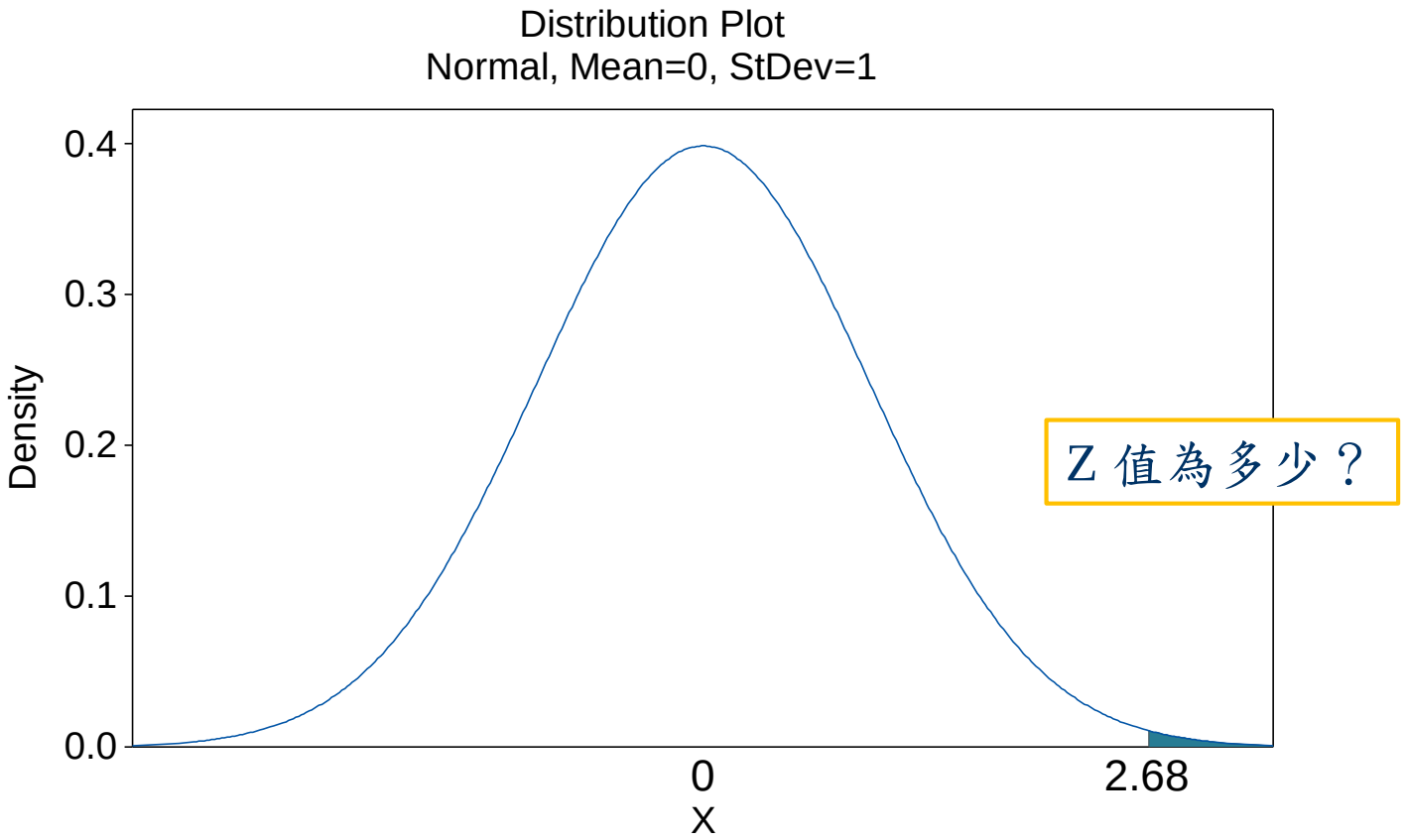
附表 4　標準常態分配 (續)

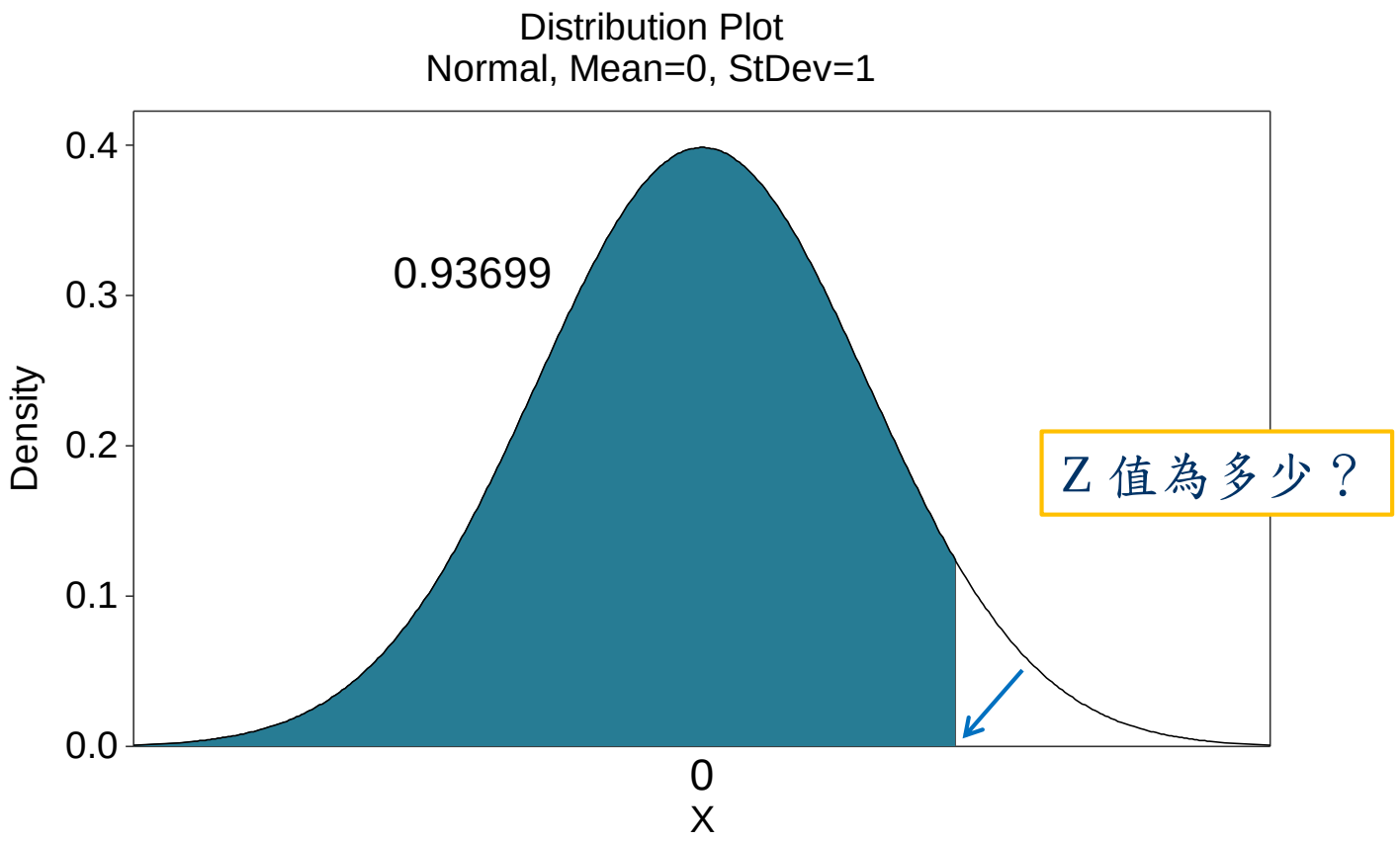
z	0.05	0.06	0.07	0.08	0.09	z
0.0	0.51994	0.52392	0.52790	0.53188	0.53586	0.0
0.1	0.55962	0.56356	0.56749	0.57142	0.57534	0.1
0.2	0.59871	0.60257	0.60642	0.61026	0.61409	0.2
0.3	0.63683	0.64058	0.64431	0.64803	0.65173	0.3
0.4	0.67364	0.67724	0.68082	0.68438	0.68793	0.4

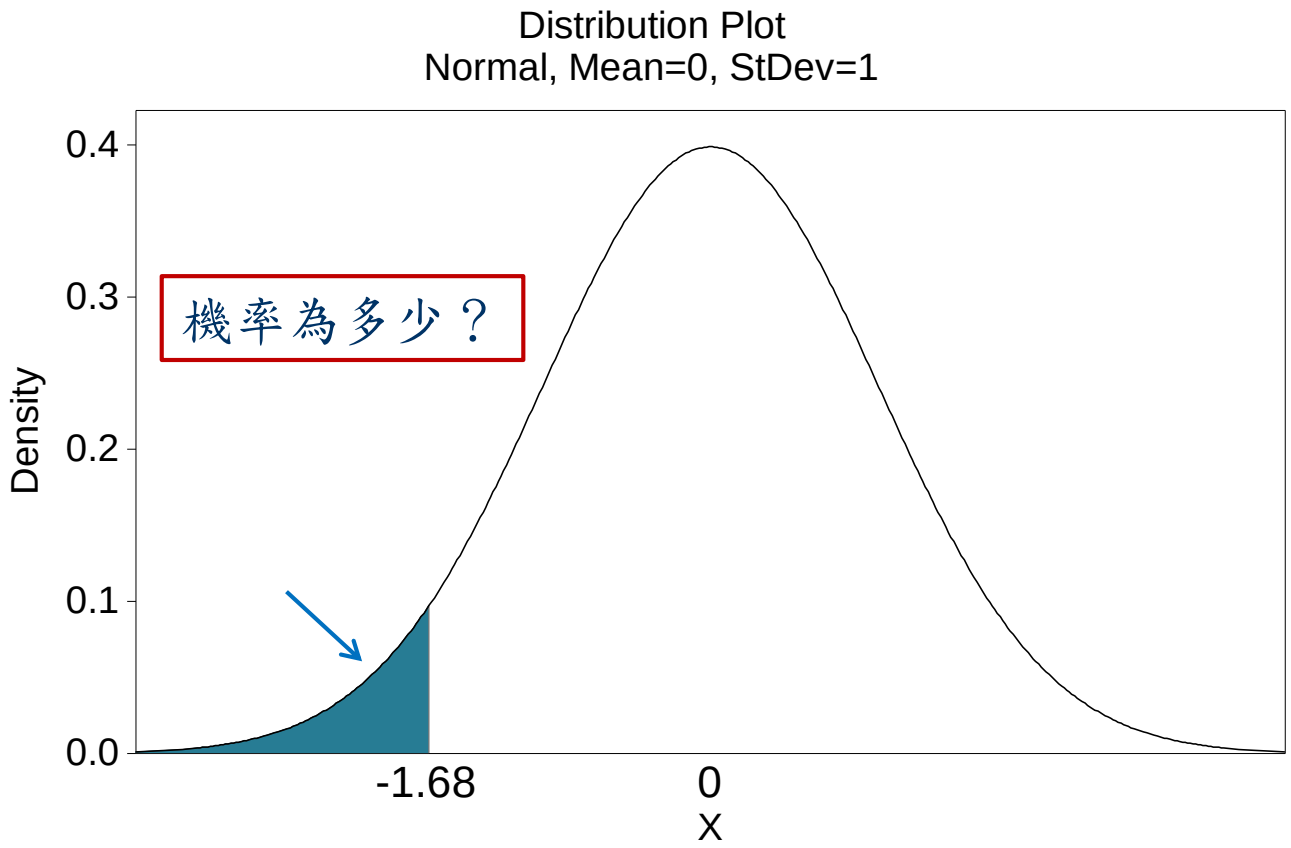
$Z = 0.45$




Distribution Plot
Normal, Mean=0, StDev=1
Density
0.4
0.3
0.2
0.1
0.0
0
2.37
X
機率為多少？


Distribution Plot
Normal, Mean=0, StDev=1
Density
0.4
0.3
0.2
0.1
0.0
0
2.68
X
Z 值為多少？


Distribution Plot
Normal, Mean=0, StDev=1
0.93699
Density
0.4
0.3
0.2
0.1
0.0
0
X
Z 值為多少？

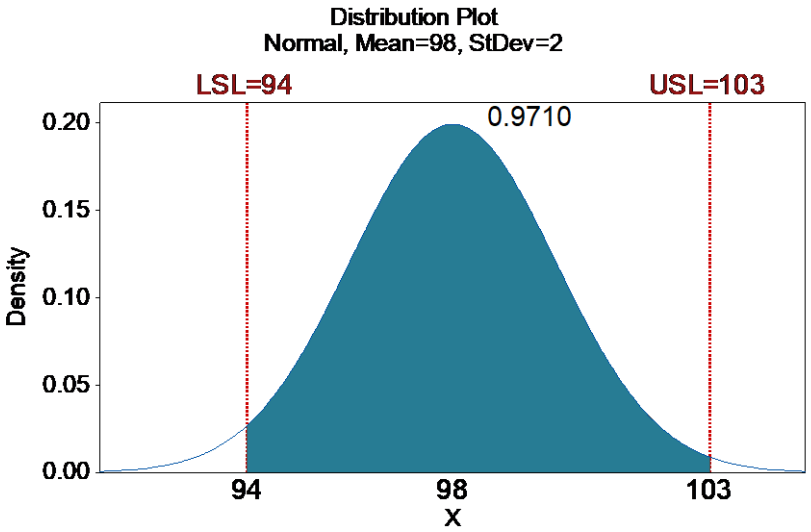

Distribution Plot
Normal, Mean=0, StDev=1
Density
0.4
0.3
0.2
0.1
0.0
機率為多少？
-1.68
0
X

應用

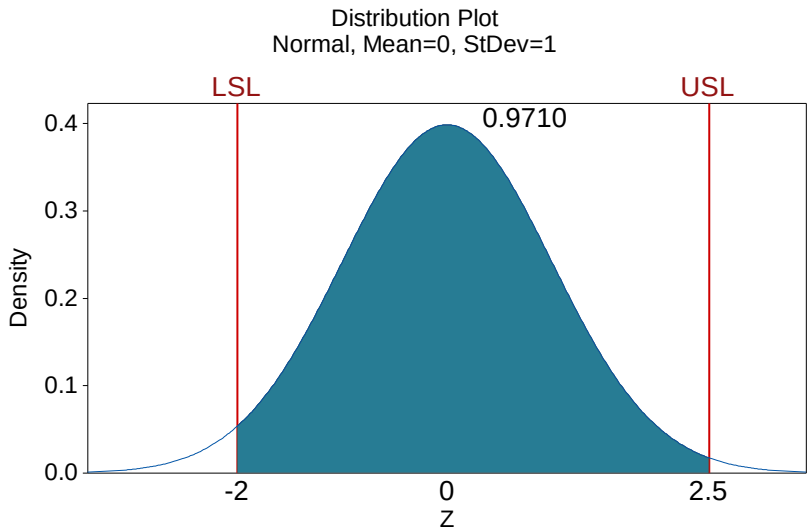
- 假設某一品質特性之規格界限 USL (upper specification limit) 為 103，LSL (lower specification limit) 為 94，由製程收集到之資料，估計得到平均數為 $\hat{\mu} = 98,\ \hat{\sigma} = 2$。如何估計不合格率?
- $\hat{\mu}$ 和 $\hat{\sigma}$ 可以在製程穩定時，從管制圖之相關資訊得到。

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \qquad \hat{\sigma} = \frac{\bar{S}}{c_4} \qquad \hat{\sigma} = \frac{\overline{MR}}{d_2}$$

應用



以原始尺度表示



以 Z 尺度表示

$$Z_U = \frac{USL - \mu}{\sigma} = \frac{103 - 98}{2} = 2.5$$

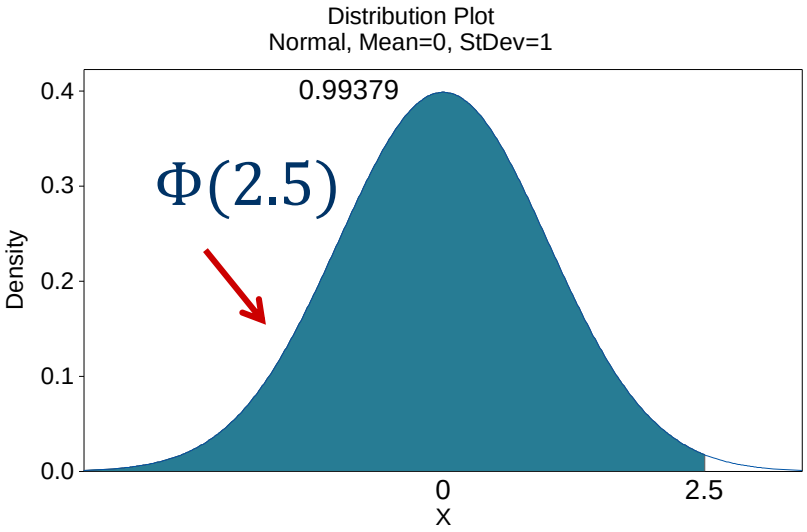
$$Z_L = \frac{LSL - \mu}{\sigma} = \frac{94 - 98}{2} = -2$$

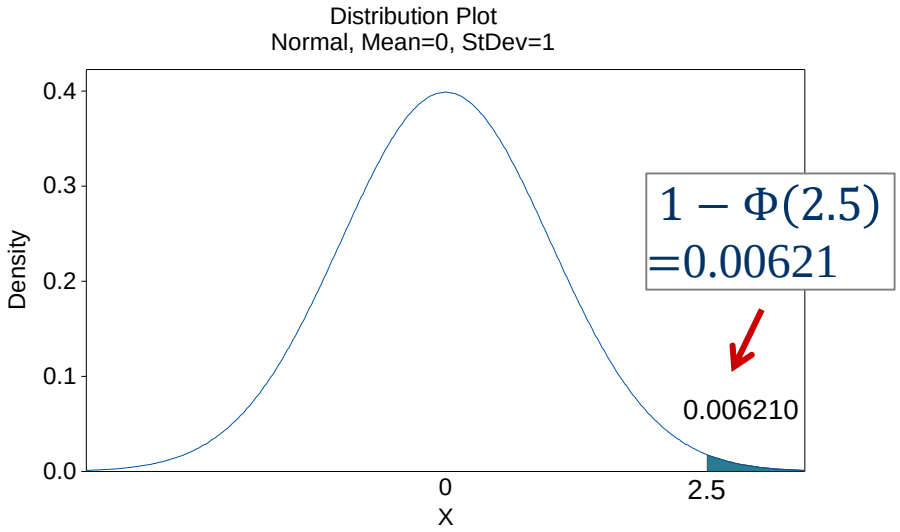
附表 4　標準常態分配

z	0.00	0.01	0.02	0.03	0.04	z
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	2.0
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	2.1
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	2.2
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	2.3
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	2.4
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	2.5
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	2.6
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	2.7
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	2.8
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	2.9

一般表格為 Φ(Z) (Z 為正號，極少數教科書會給正、負號)

應用

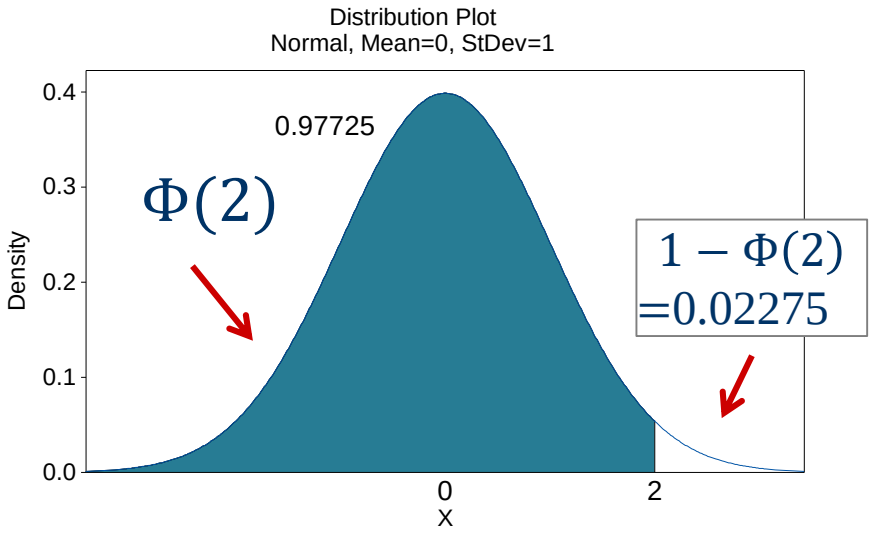


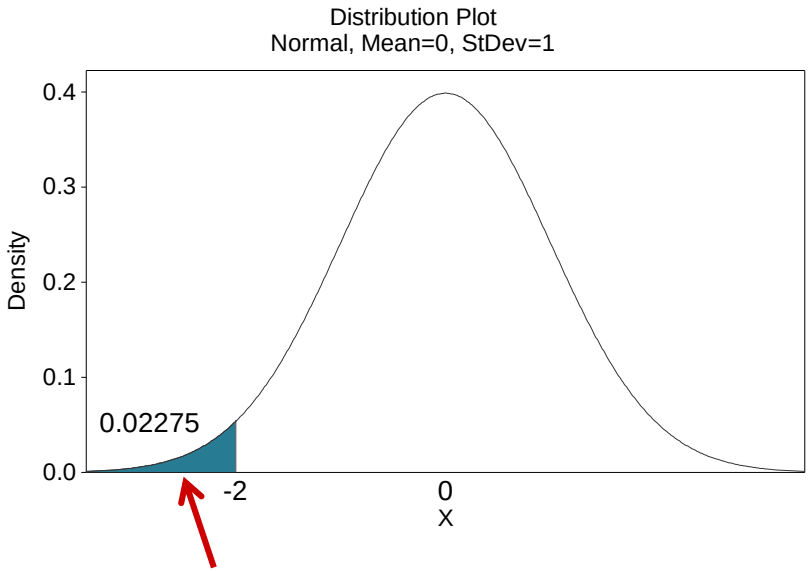


以超出 USL 之不合格率為例，我們必須先查表求 Φ(2.5) (不超出上限 USL之機率)，1 – Φ(2.5) 才是超出 USL 之機率。

落在上限 USL 外之機率

不合格率總和為=0.00621+0.02275=0.02896
合格度為 1 − 不合格率=1-0.02896-0.97104





落在下限 LSL 外之機率

由於一般表中之 Z 值為正號，若要估計超出 LSL之機率，相當於計算 Φ(−2)，但表中 Z 值為正號，因此需先計算 1 − Φ(2)。由於常態分配為對稱，Φ(−2) = 1 − Φ(2)，見上圖之說明。

應用

- 由上述範例可看出，當 $|Z|$ 值越大時，代表製程平均數離規格界限越遠，不合格率也越低。Z 值也可以作為製程之績效指標。
- 上、下限之 Z 值分別寫成 Z_U 和 Z_L，定義為：

$$Z_U = \frac{USL-\mu}{\sigma}$$

$$Z_L = \frac{\mu-LSL}{\sigma}$$

- 一般情況 (指 μ 落在規格界限內) 下，Z_U 與 Z_L 均為正值。Z_U 和 Z_U 也稱為 Sigma Level。

應用

- 請注意，一般Z值定義為：

$$Z = \frac{X-\mu}{\sigma}$$

- 將 USL 代入 X 會得到正值的 Z，LSL 代入公式 X會得到負值的 Z。
- 若有上、下限，則 Z_U 和 Z_L 必須整合成單一 Z 值 (稱為 Z_{Bench})，以方便比較。

應用

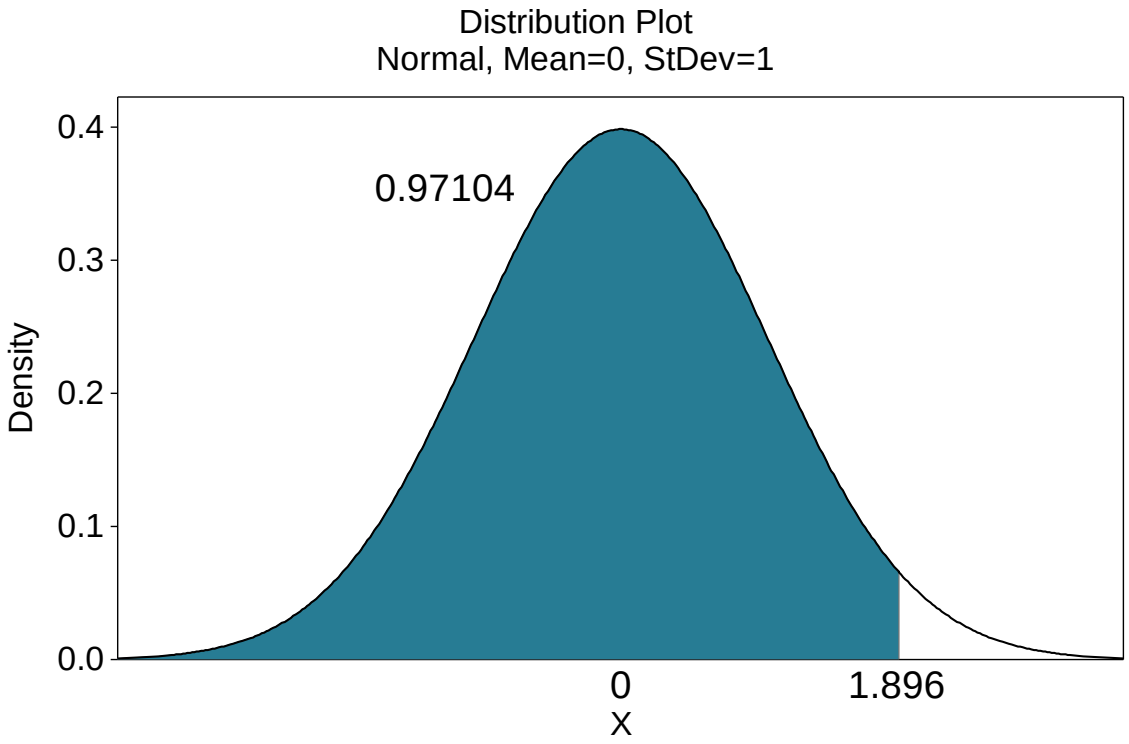
- 首先計算整體不合格率 p_{total}，$p_{total} = p_{\mathrm{U}} + p_{\mathrm{L}} = 0.02896$
- 利用 Φ 之反函數可得 $Z_{Bench} = \Phi^{-1}(1 - p_{total})$
- 以本例而言，合格率為 0.97104，
  利用反函數 $\Phi^{-1}(1 - 0.02896) = \Phi^{-1}(0.97104)$，
  可得 Z (也就是 Z_{Bench}) 為 1.896。
- 注意：$Z_{Bench} < \min\{Z_{\mathrm{U}}, Z_{\mathrm{L}}\}$

應用

$\Phi(1.89) = 0.97062$

1.8	0.96407	0.96485	0.96562	0.96637	0.96711	1.8	1.8	0.96784	0.96856	0.96926	0.96995	0.97062	1.8
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	1.9	1.9	0.97441	0.97500	0.97558	0.97615	0.97670	1.9

$\Phi(1.90) = 0.97128$



t 分配

附表 6　t 分配表

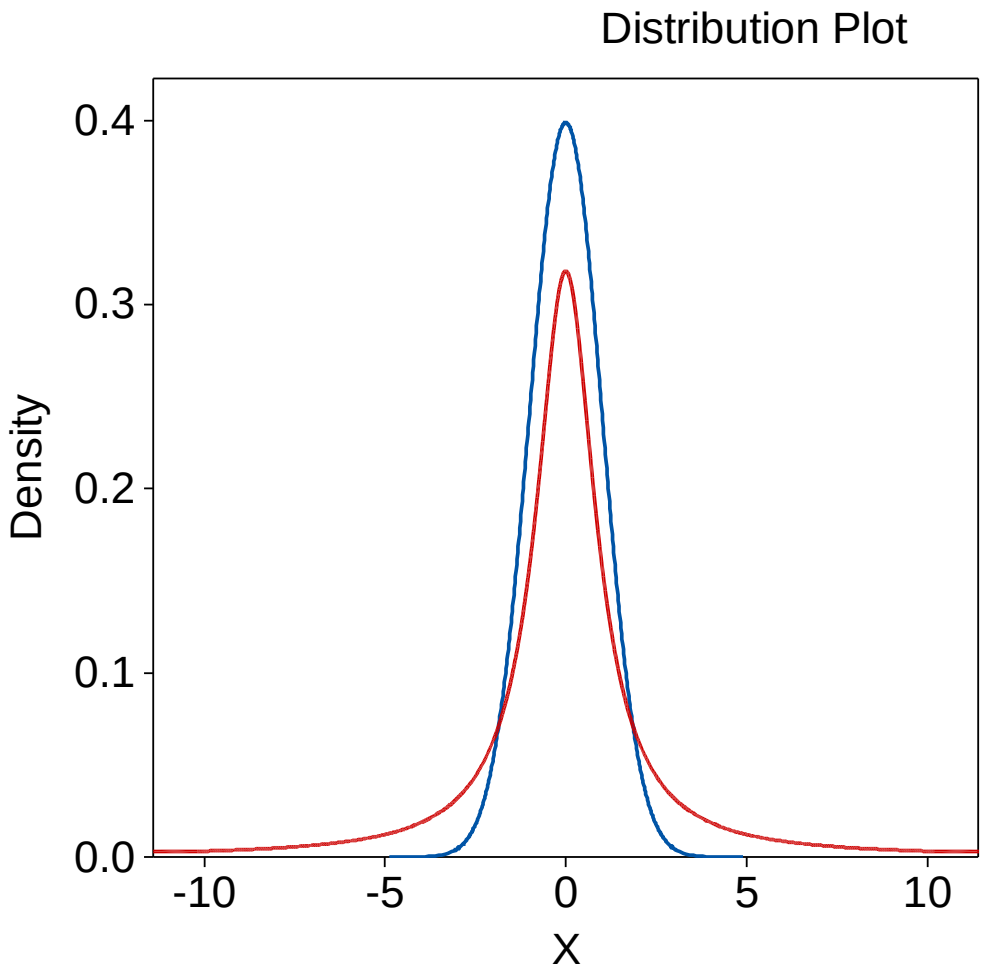
	α									
v	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.32	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073

t 分配

附表 6　t 分配表

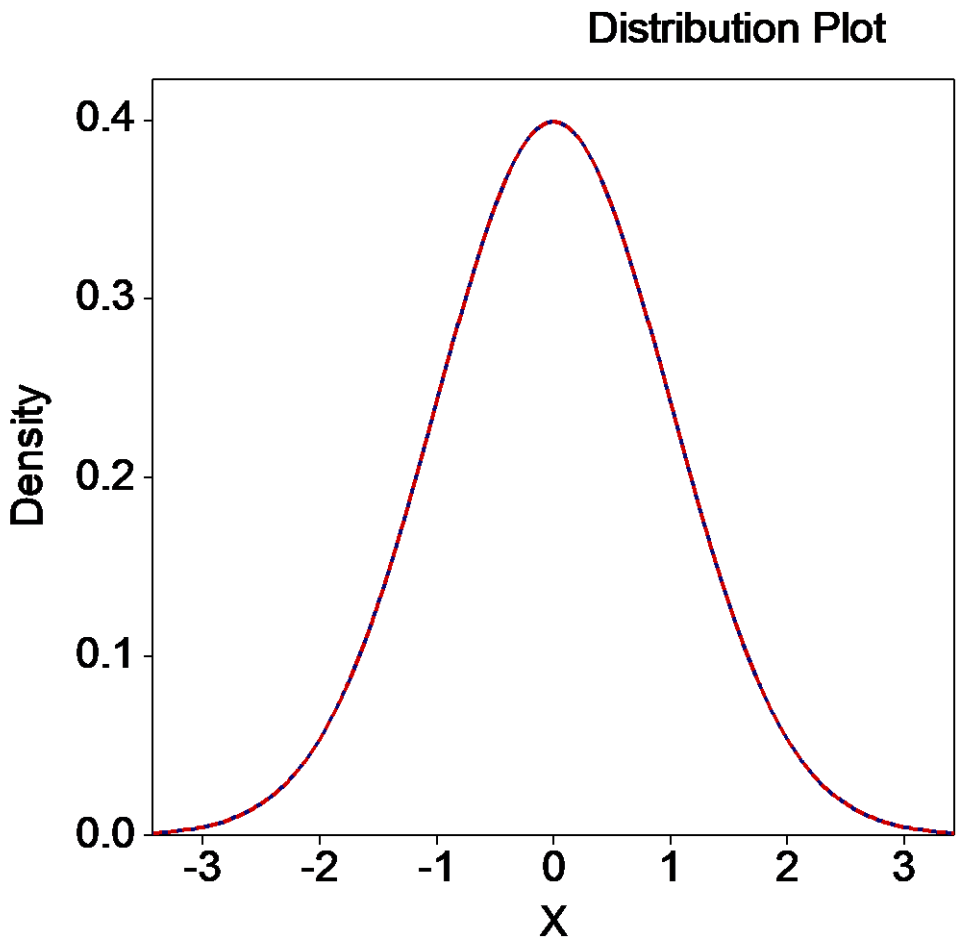
	α									
ν	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

t 分配 vs. 標準常態分配



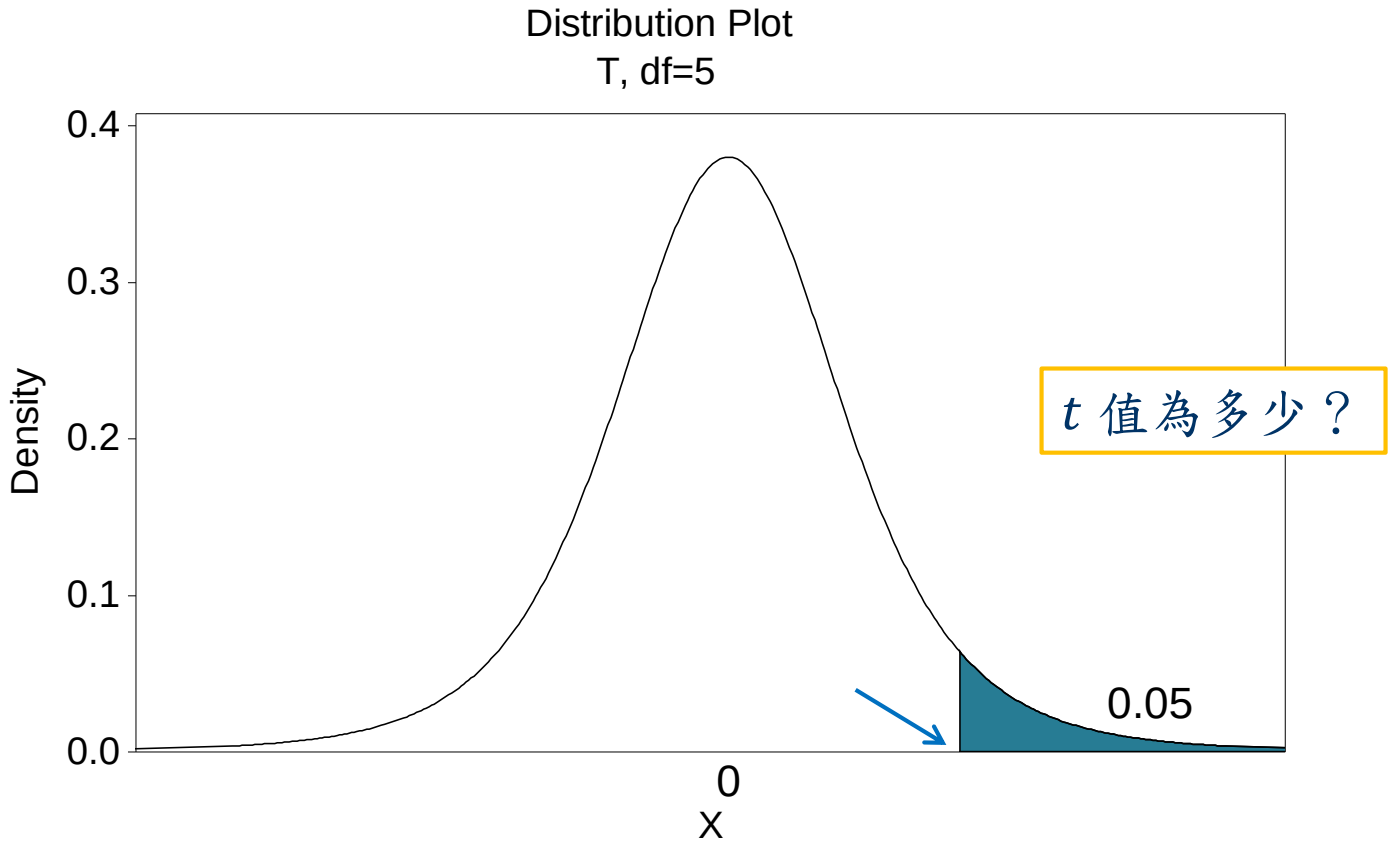
t 分配是一個厚尾之分配 (heavy tail)

t 分配 vs. 標準常態分配

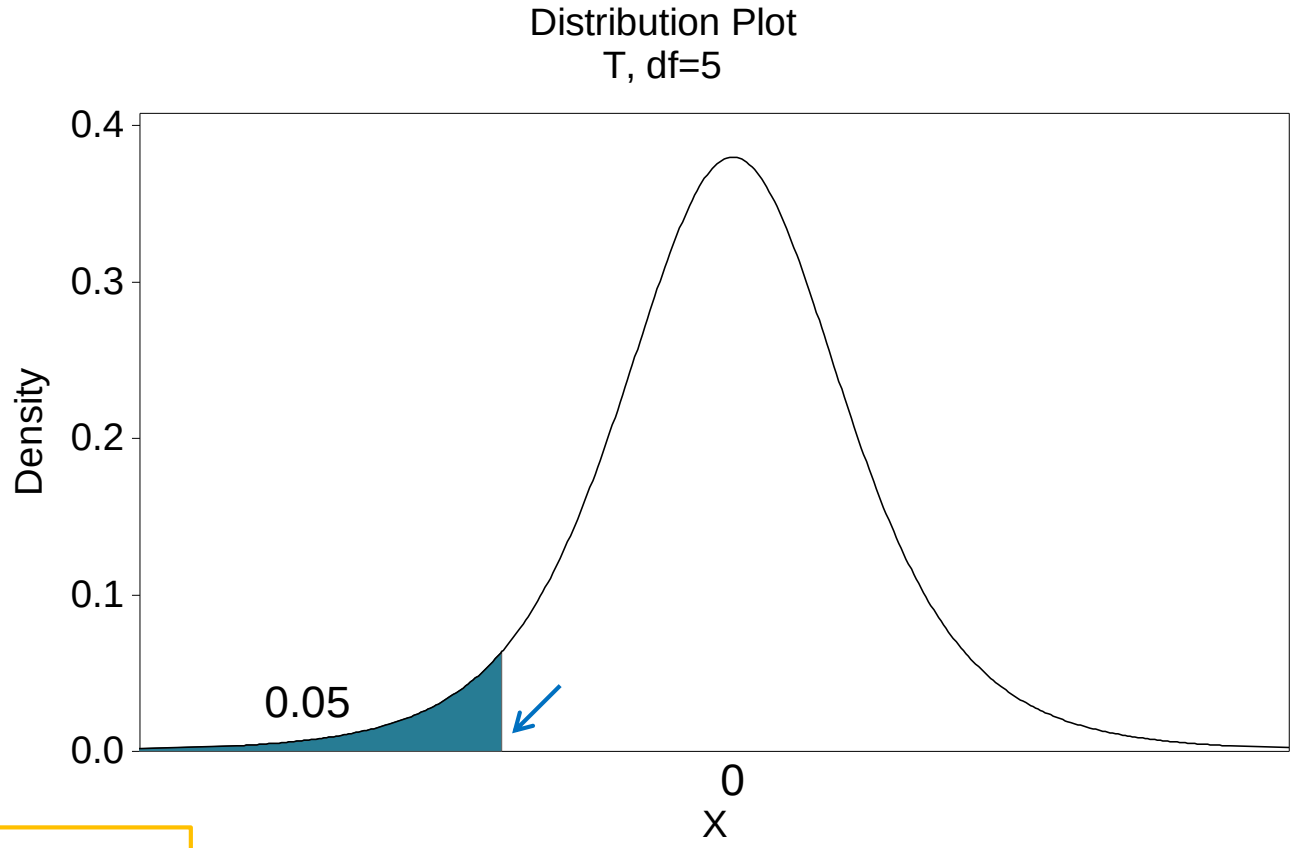


當 degrees of freedom 自由度趨近無限大時，則 t 分配近似於標準常態分配。

t 分配



t 分配



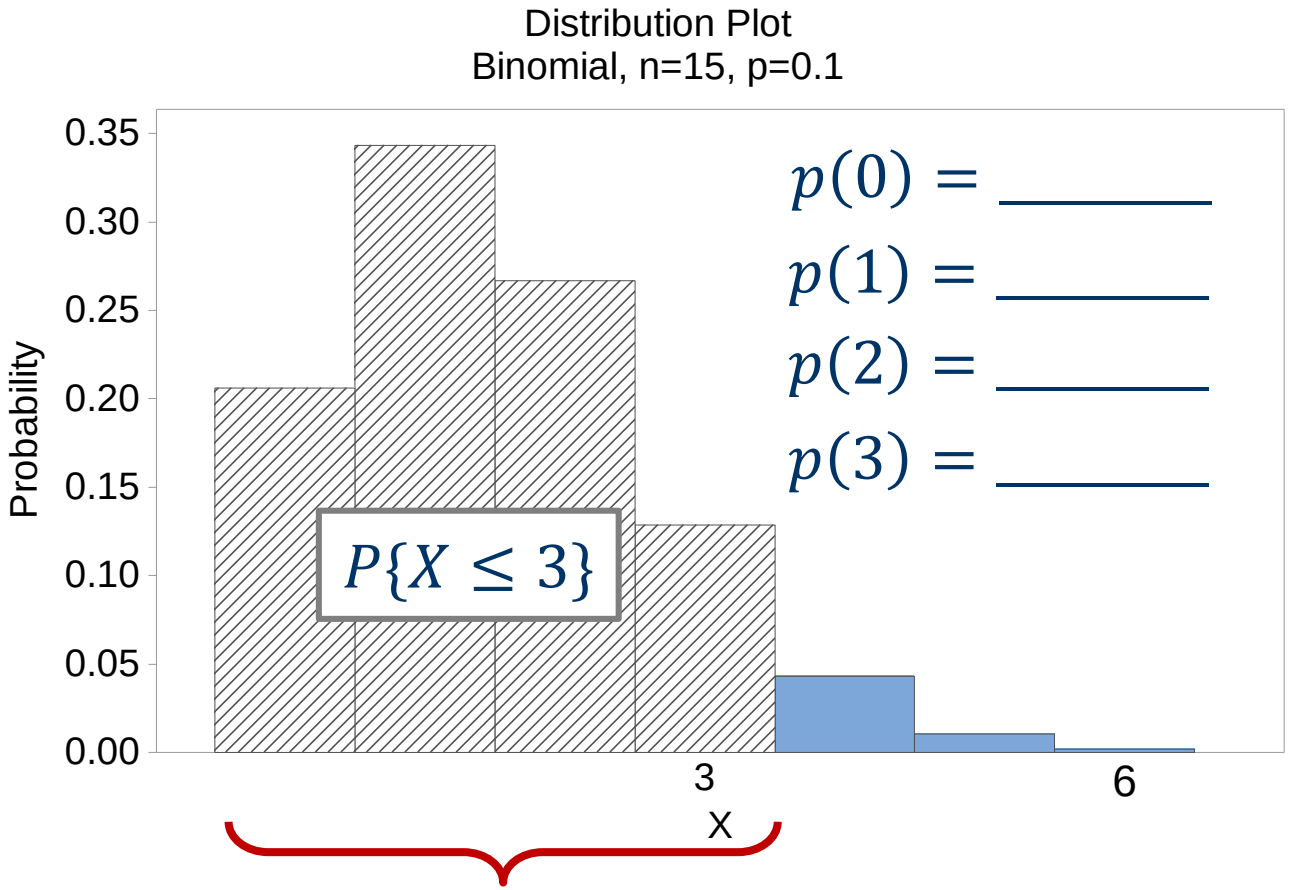
t 值為多少？

二項 (binomial) 分配

附表 2　累積二項分配 (續)

		p									
n	x	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
15	0	.463	.206	.087	.035	.013	.005	.002	.000	.000	.000
	1	.829	.549	.319	.167	.080	.035	.014	.005	.002	.000
	2	.964	.816	.604	.398	.236	.127	.062	.027	.011	.004
	3	.995	.944	.823	.648	.461	.297	.173	.091	.042	.018
	4	.999	.987	.938	.836	.686	.515	.352	.217	.120	.059
	5	1.000	.998	.983	.939	.852	.722	.564	.403	.261	.151
	6		1.000	.996	.982	.943	.869	.755	.610	.452	.304
	7			.999	.996	.983	.950	.887	.787	.654	.500
	8			1.000	.999	.996	.985	.958	.905	.818	.696
	9				1.000	.999	.996	.988	.966	.923	.849
	10					1.000	.999	.997	.991	.975	.941
	11						1.000	1.000	.998	.994	.982
	12								1.000	.999	.996
	13									1.000	1.000
16	0	.440	.185	.074	.028	.010	.003	.001	.000	.000	.000
	1	.811	.515	.284	.141	.063	.026	.010	.003	.001	.000
	2	.957	.789	.561	.352	.197	.099	.045	.018	.007	.002
	3	.993	.932	.790	.598	.405	.246	.134	.065	.028	.011
	4	.999	.983	.921	.798	.630	.450	.289	.167	.085	.038
	5	1.000	.997	.976	.918	.810	.660	.490	.329	.198	.105

二項 (binomial) 分配



$$P\{X \le 3\} = p(0) + p(1) + p(2) + p(3)$$

卜瓦松 (Poisson) 分配

附表 3　累積卜瓦松分配

				λ				
x	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60
0	0.990	0.951	0.905	0.819	0.741	0.670	0.607	0.549
1	1.000	0.999	0.995	0.982	0.963	0.938	0.910	0.878
2		1.000	1.000	0.999	0.996	0.992	0.986	0.977
3				1.000	1.000	0.999	0.998	0.997
4						1.000	1.000	1.000
x	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40
0	0.497	0.449	0.407	0.368	0.333	0.301	0.273	0.247
1	0.844	0.809	0.772	0.736	0.699	0.663	0.627	0.592
2	0.966	0.953	0.937	0.920	0.900	0.879	0.857	0.833
3	0.994	0.991	0.987	0.981	0.974	0.966	0.957	0.946
4	0.999	0.999	0.998	0.996	0.995	0.992	0.989	0.986
5	1.000	1.000	1.000	0.999	0.999	0.998	0.998	0.997
6				1.000	1.000	1.000	1.000	0.999
7								1.000
x	1.50	1.60	1.70	1.80	1.90	2.00	2.20	2.40
0	0.223	0.202	0.183	0.165	0.150	0.135	0.111	0.091
1	0.558	0.525	0.493	0.463	0.434	0.406	0.355	0.308
2	0.809	0.783	0.757	0.731	0.704	0.677	0.623	0.570
3	0.934	0.921	0.907	0.891	0.875	0.857	0.819	0.779
4	0.981	0.976	0.970	0.964	0.956	0.947	0.928	0.904
5	0.996	0.994	0.992	0.990	0.987	0.983	0.975	0.964
6	0.999	0.999	0.998	0.997	0.997	0.995	0.993	0.988
7	1.000	1.000	1.000	0.999	0.999	0.999	0.998	0.997
8				1.000	1.000	1.000	1.000	0.999
9								1.000

卜瓦松 (Poisson) 分配

附表 3　累積卜瓦松分配

					λ			
x	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60
0	0.990	0.951	0.905	0.819	0.741	0.670	0.607	0.549
1	1.000	0.999	0.995	0.982	0.963	0.938	0.910	0.878
2		1.000	1.000	0.999	0.996	0.992	0.986	0.977
3				1.000	1.000	0.999	0.998	0.997
4						1.000	1.000	1.000
x	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40
0	0.497	0.449	0.407	0.368	0.333	0.301	0.273	0.247
1	0.844	0.809	0.772	0.736	0.699	0.663	0.627	0.592
2	0.966	0.953	0.937	0.920	0.900	0.879	0.857	0.833
3	0.994	0.991	0.987	0.981	0.974	0.966	0.957	0.946
4	0.999	0.999	0.998	0.996	0.995	0.992	0.989	0.986
5	1.000	1.000	1.000	0.999	0.999	0.998	0.998	0.997

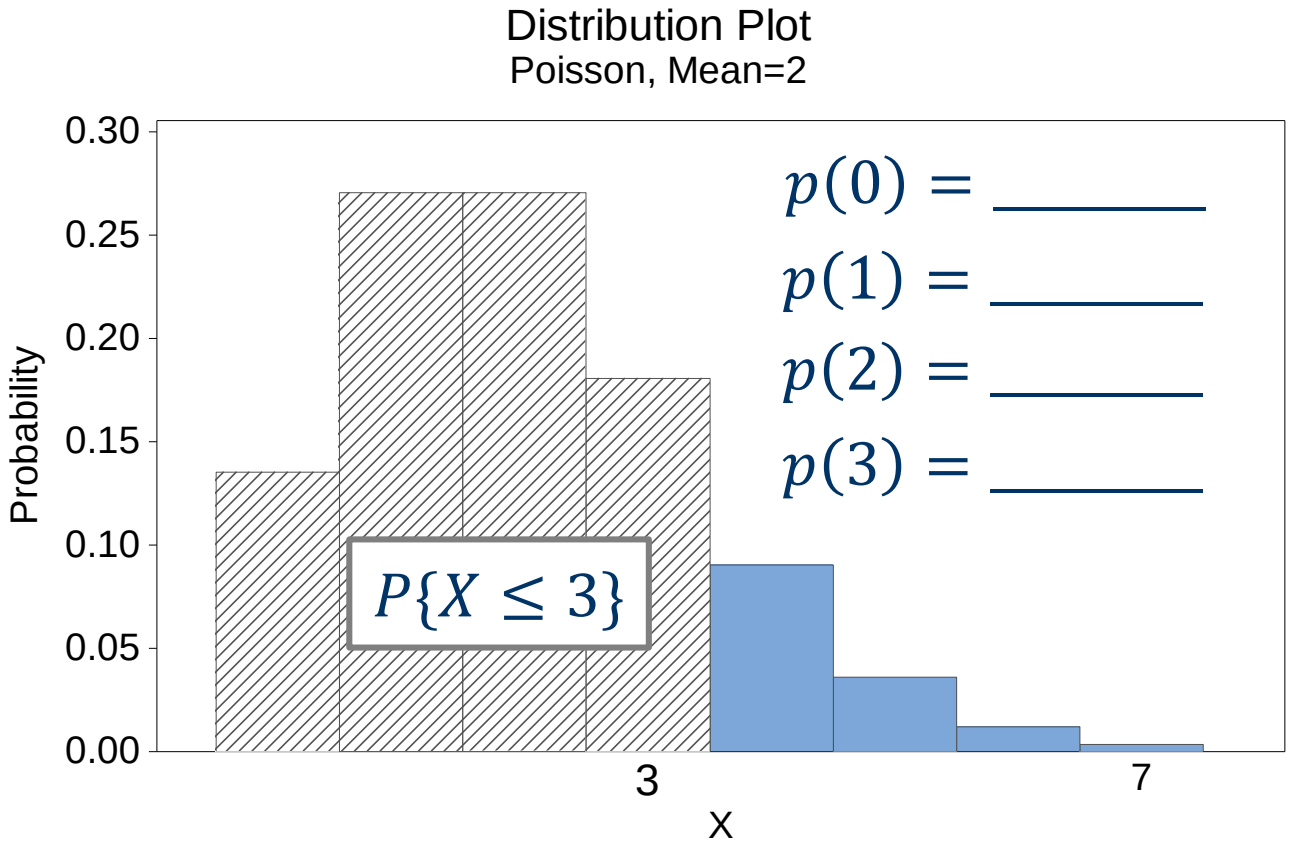
$\lambda = 0.8$

x	p(x)	cum. p(x)
0	0.449	0.449
1	0.359	0.809
2	0.144	0.953
3	0.038	0.991
4	0.008	0.999
5	0.001	1.000

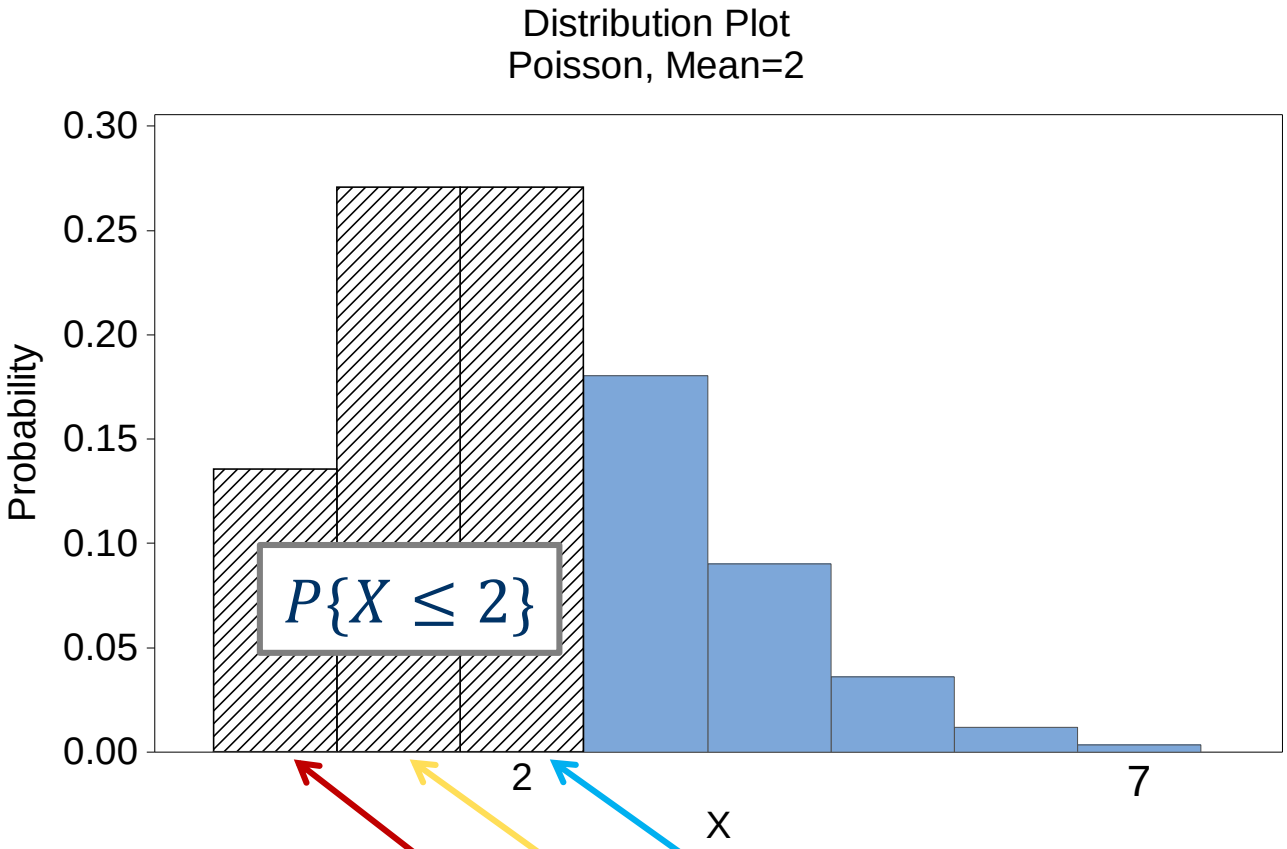
$$P\{X \le 1\} = p(0) + p(1) = 0.449 + 0.359 = 0.809$$

$$P\{X \le 2\} = p(0) + p(1) + p(2) = 0.953$$

卜瓦松 (Poisson) 分配



卜瓦松 (Poisson) 分配



$$P\{X \leq 2\} = p(0) + p(1) + p(2)$$